University of Pardubice
Faculty of Transport Engineering

# DISTRIBUTION PROBLEMS IN LOGISTICS SYSTEMS

DOCTORAL THESIS STATEMENTS

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# 1 State of the Art

Logistics systems ensure the smooth and efficient flow of materials, goods, and information between manufacturers, suppliers, and customers, and constitute an integral part of the modern economy. Their proper functioning represents a crucial factor in companies' competitiveness, with the optimization of distribution problems playing a key role in this regard [1].

Distribution problems can address the optimal placement, routing, and transportation of consignments. Their optimization has a direct impact on operational efficiency and also helps reduce fuel consumption and emissions, contributing to sustainable development [2].

At present, greenhouse gas emissions have become a serious problem, with the transport industry being one of their most significant sources. As a result, research in both industry and academia is increasingly focused on greener transportation [3].

To reduce pollution caused by the use of petroleum-based fuels, companies have begun introducing alternative fuel vehicles (AFVs) into their fleets in response to regulatory measures. AFVs use more environmentally friendly energy sources such as biodiesel, ethanol, methanol, hydrogen, natural gas, propane, and electricity.

Among AFVs, electric vehicles (EVs) are particularly attractive. However, their broader adoption is still limited due to operational challenges faced by companies using EVs. The main limitations of EVs compared to internal combustion engine vehicles include: limited driving range [4, 5], significantly longer charging times [6], and insufficient availability of charging infrastructure [6, 7, 8].

Given these limitations, the deployment of EVs requires the use of efficient routing and energy replenishment strategies. A key element is the design of energy-efficient routes and the identification of optimal charging or swapping stations (SDPs) for energy replenishment along the vehicle's route [9]. Efficient EV routing plays a crucial role in promoting their use [10].

New methods for EV routing and energy replenishment decisions can significantly support the broader deployment of EVs in customer transportation services due to higher computational efficiency, better route planning, and EV charging decision-making. This would allow for greater representation of EVs in the fleets of logistics companies and contribute to the achievement of environmental objectives in transportation.

## 1.1 Vehicle Routing Problem with Refueling

The problem in the field of distribution planning that focuses on vehicle routing with simultaneous refueling or energy replenishment is known as the Vehicle Routing Problem with Refueling (VRPR). [11]

This problem represents an extension of the basic variant of the Vehicle Routing Problem, which takes into account constraints related to the vehicle's maximum driving range when planning cyclic routes. This constraint requires the inclusion of SDPs in the routes so that the vehicle does not run out of fuel or energy during its journey, while the value of the optimization criterion remains minimal [12].

Given that the VRPR is an NP-hard problem and more complex instances therefore cannot be solved in polynomial time using exact methods, approximation approaches are more commonly employed, which only search for suboptimal solutions in polynomial time [8]. The most frequently used approaches are metaheuristic methods [13].

Single-phase and two-phase approaches can be applied to solve the VRPR using approximation methods.

Single-phase approaches assign SDPs to a circular route directly during its construction. The disadvantage of these approaches lies in the higher complexity of the applied method, which must simultaneously determine the sequence of visits to serviced nodes and the assignment of SDPs to the circular route. The higher complexity leads to longer calculation times or the need for a higher number of iterations [14, 15].

The two-phase approach addresses the VRPR in two stages. The first stage is used to construct the sequence of serviced nodes (customers). In the second stage, SDPs are assigned to the route with an already determined sequence of serviced nodes. In the literature, this approach appears in various variants applied to different forms of the VRPR.

If a metaheuristic method or another iterative procedure is used to solve the first phase, the method for solving the second phase can be applied to all sequences of serviced nodes generated in individual iterations. On the one hand, this strategy increases computational complexity. On the other hand, it increases efficiency because it allows the proposed routes to be evaluated in each iteration, taking into account the need for refueling. Many methods in the literature implicitly adopt this strategy [16].

## 1.2 Sequence Gas Station Problem

The problem whose purpose, within a two-phase approach to solving the VRPR, is to assign SDPs to a route with a fixed sequence of serviced nodes can be referred to as the Sequence Gas Station Problem (SGSP). [17] The problem can be defined as a variant of the shortest path problem in a graph (SPP), whose objective is to find the shortest path from the start node to the end node that includes all serviced nodes in the prescribed order, as well as appropriate SDPs, such that the route is feasible with respect to the vehicle's limited driving range.

The SGSP presented in this dissertation can be formulated, based on Montoya et al. [18], in a manner that allows the problem to be solved using exact methods that are commonly used to solve SPP in polynomial time. Based on testing conducted by Głabowski et al. [19], it can be stated that in terms of computation time, the most effective methods suitable for solving the SGSP are: the basic variant of Dijkstra's algorithm, the Small Label First (SLF) algorithm, and the Large Label Last (LLL) algorithm. Among the efficient algorithms applicable to the SGSP, the A* algorithm can also be included, as it typically expands fewer nodes than the classical Dijkstra algorithm, leading to faster solution times.

In the area of heuristic methods, two approaches are most commonly applied to solve the SGSP. The first is based on the work of Montoya et al. [20] and involves inserting SDPs into the route based on minimizing the energy deficit along the route. The second is based on inserting SDPs into the route if the amount of fuel does not allow another customer on the route to be visited. In this case, one of the most comprehensive solution methods is provided by Zhang et al. [21].

## 1.3 Critical Evaluation of the State of the Art

The chapter *State of the Art* analyzed distribution problems with the aim of identifying opportunities for optimizing vehicle routing methods and fuel or energy replenishment planning. Based on this analysis, the following conclusions were formulated:

1. A potential area for improvement lies in the SGSP, which is addressed within the second phase of two-phase approaches to solving the VRPR.

2. Two-phase approaches to solving the VRPR that use iterative methods to solve the first phase require solving a large number of SGSPs in the second phase, which places high demands on the speed of the method used.
3. A large proportion of studies address the SGSP using heuristic methods, which enable rapid identification of suboptimal solutions, but do not guarantee optimality or finding a solution, even if it exists, which can negatively affect the VRPR solution.
4. Studies that employ exact methods to solve the SGSP in most cases address a variant that constitutes an NP-hard problem.
5. Methods for solving the SPP, which enable finding the optimal solution to the problem in polynomial time, can be used to solve the SGSP variant presented in the dissertation and some of its modifications.
6. No methods or modifications of the methods mentioned in point 5 were identified that exploit the specific properties of the SGSP to optimize computational time.

Based on the above conclusions, it can be stated that the potential for improving the solution of the SGSP and the related VRPR lies in the development of a new exact method that exploits the specific properties of the SGSP, in particular the fixed order of mandatory visited nodes, to reduce computational time. Its application within two-phase approaches to solving the VRPR would lead to a reduction in computation time compared to exact methods based on the SPP and, at the same time, to an improvement in the value of the optimization criterion compared to heuristic methods.

The new method could be applied within two-phase approaches to solving the VRPR even for more complex problems, where existing exact methods for solving the SGSP are time-inefficient. For such problems, the method would enable route evaluation in individual iterations based on the optimal assignment of SDPs, which, compared to heuristic methods used, would lead to improved accuracy in the design of cyclic routes and would guarantee finding a solution to the SGSP whenever one exists.

Due to the optimal assignment of SDPs to routes, the application of the new method would contribute to alleviating EV operational constraints, thereby promoting their wider use.

# 2 Research Objectives

The objective of the dissertation is to propose a new exact method for solving a distribution problem focused on planning energy replenishment stops in EV routing. The purpose is to create a method that enables the identification of an optimal charging or battery swapping plan in a shorter computation time than existing exact approaches.

Based on the analysis of the state of the art, the main objective of the dissertation was specified as the development of a new exact method that enables finding the optimal solution of the SGSP in a shorter computation time than existing exact methods. Owing to its higher computational efficiency, this method will also be applicable to solving VRPR instances whose complexity has so far hindered the effective use of existing exact methods for the SGSP.

The dissertation will contribute to improving EV routing and planning of stops for recharging, thereby mitigating the operational limitations of this type of vehicle. The results of the work will thus support the wider use of EVs in logistics operations and contribute to the fulfillment of environmental goals in transport. The dissertation will also expand theoretical knowledge in operational research related to the optimization of EV-focused distribution problems.

To achieve the objective of the dissertation, it is necessary to:

1. Analyze the current state and approaches to solving distribution problems focused on vehicle routing and fuel or energy replenishment.
2. Identify key requirements and potential optimization opportunities for solving the studied distribution problem.
3. Propose a new method for solving the studied distribution problem.
4. Implement the proposed method and verify its effectiveness in terms of computation time and optimization criterion value.

# 3 Research Methodology

The methodological procedure of the dissertation establishes a comprehensive framework for proposing a new exact method for solving the distribution problem focused on planning energy replenishment stops within EV routing, its subsequent evaluation in terms of computational time and optimization criterion, and the verification of its practical applicability. The diagram of the methodological procedure steps is shown in Figure 1.

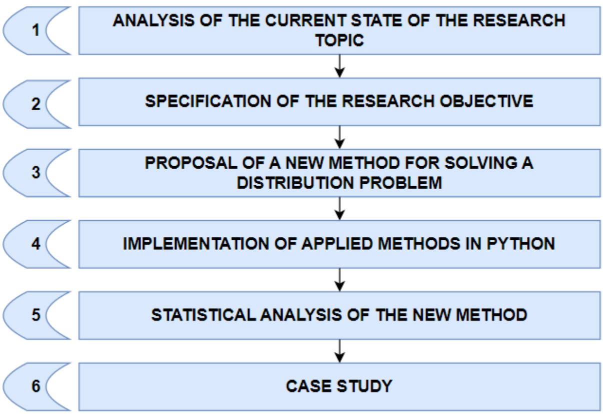


Figure 1. Methodological Procedure

**The first step** involves an analysis of the state of the art in the field of distribution problems, with a focus on vehicle routing and the planning of fuel or energy replenishment along routes. Within this step, existing approaches to solving these problems are examined, and the requirements and constraints considered in their solution are identified. Based on the identified shortcomings of current methods, potential opportunities for optimizing the solution of the studied problems are subsequently defined, providing the basis for proposing a new exact solution method.

**The second** step builds on the conclusions of the state-of-the-art analysis and, based on them, specifies the objective of the dissertation. In this dissertation, the objective is defined as **the development of a new exact method that enables finding the optimal solution of the SGSP in a shorter computation time than existing exact methods.**

**The third step** consists of proposing a new method for solving the studied distribution problem. For this purpose, the problem is formulated using graph theory, which serves as the basis for describing both the problem structure and the main principles of the proposed method. Pseudocode is used to describe the individual algorithms of the new method.

**The fourth step** focuses on the implementation of all methods used in the subsequent stages of the dissertation in the Python programming language. This creates a unified computing environment that allows the application of individual methods in solving specific practical problems and subsequent comparison of their effectiveness in terms of the monitored criteria.

**The fifth step** is devoted to evaluating the effectiveness of the new method in terms of computational time, optimization objective value, and problem solvability. For this purpose, the new method is compared with existing exact and heuristic methods. The comparison is conducted at the level of both the SGSP and the VRPR using sample sets consisting of randomly generated problem instances with different levels of complexity, labeled as “Low,” "Medium," and "High." The methods used for the comparison are those specified in the first step of the methodological procedure and shown in Figure 2.

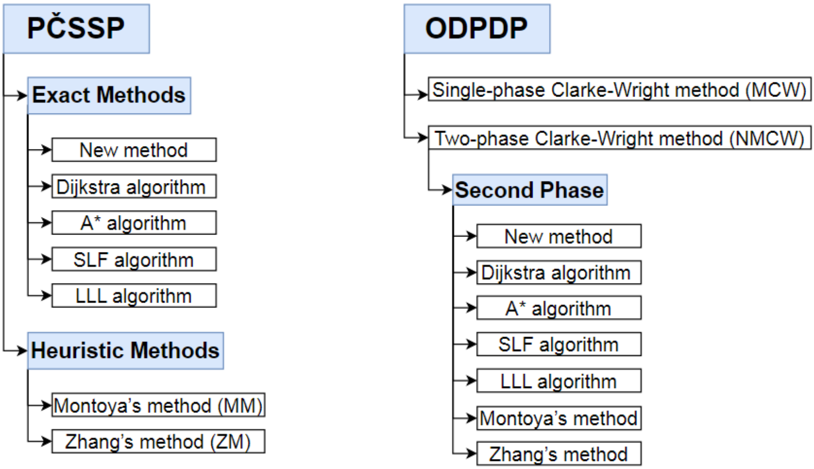


Figure 2. Compared methods

Methods and indicators of statistical analysis are used to evaluate the results of the comparison. The main ones include:

- **paired statistical tests** – verification that the new method achieves statistically significantly lower computation time and optimization criterion value compared to the methods being compared,
- **confidence intervals** – estimation of the average reduction in computation time and the value of the optimization criterion achieved by applying the new method compared to the comparative method across the entire population of the examined problem type,
- **simple individual indices** – expression of relative change in average computation time and optimization criterion value at individual problem complexity levels relative to the lowest complexity level.

**The sixth step** focuses on demonstrating the practical application of the new method using a case study focused on solving a real-world VRPR. Cyclic routes for EVs, including stops for recharging, are designed using a two-phase Clarke–Wright method in combination with the new method, as well as in combination with selected exact and heuristic methods for solving the SGSP that were identified as the most effective. The results are compared primarily in terms of computational time and the value of the optimization criterion.

# 4 Achieved Results

The effectiveness of the new method was evaluated using statistical analysis focused on computational time, optimization criterion value, and problem solvability. Its practical applicability was demonstrated through a case study.

## 4.1 Statistical Analysis

The evaluation of the achieved results is primarily based on paired statistical tests, while Tables 1–4 provide a complementary overview of statistical indicators enabling quantitative comparison of the methods compared. The tables show the following indicators: lower and upper bounds of confidence intervals (Gdr, Ghr), simple individual indices (SII), and the number of instances for which a given method did not find a solution (Unsolved).

Using paired statistical tests, it was demonstrated that the new method achieves statistically significantly lower computational time than all compared exact methods across all levels of problem complexity, both when solving the SGSP and when applied within the solution of the VRPR.

Table 1 presents the values of the statistical indicators achieved in terms of computational time when comparing the new method with exact methods in solving the SGSP.

Table 1 Comparison of SGSP Solution Methods in Terms of Computation Time

| Problem complexity | | New method | Dijkstra | A* | SLF | LLL |
|---|---|---|---|---|---|---|
| **Low** | Gdr [%] | - | 99.7914 | 99.2437 | 99.8945 | 99.8935 |
| | Ghr [%] | - | 99.8224 | 99.4754 | 99.9104 | 99.9096 |
| **Medium** | SII | 4.1988 | 14.0421 | 14.8231 | 13.9477 | 14.1461 |
| | Gdr [%] | - | 99.9445 | 99.8968 | 99.9719 | 99.9721 |
| | Ghr [%] | - | 99.9509 | 99.9141 | 99.9751 | 99.9752 |
| **High** | SII | 31.4409 | 150.4120 | 157.7772 | 141.4320 | 145.7129 |
| | Gdr [%] | - | 99.9614 | 99.9383 | 99.9798 | 99.9801 |
| | Ghr [%] | - | 99.9685 | 99.9543 | 99.9832 | 99.9835 |

Table 2 presents the values of the statistical indicators obtained from the comparison of the new method with exact methods when applied within the solution of the VRPR in terms of computational time.

Table 2 Comparison of VRPR Solution Methods in Terms of Computation Time

| Problem complexity | | New method | Dijkstra | A* | SLF | LLL |
|---|---|---|---|---|---|---|
| **Low** | Gdr [%] | - | 87.0692 | 5.9636 | 93.9593 | 93.8630 |
| | Ghr [%] | - | 88.0821 | 19.6382 | 94.5411 | 94.3600 |
| **Medium** | SII | 4.3232 | 8.7866 | 3.3739 | 8.2181 | 8.8642 |
| | Gdr [%] | - | 93.7779 | 23.8276 | 96.9929 | 97.0886 |
| | Ghr [%] | - | 94.2670 | 29.8640 | 97.2352 | 97.3223 |
| **High** | SII | 24.5338 | 56.6196 | 50.3024 | 52.7924 | 54.8340 |
| | Gdr [%] | - | 94.3968 | 66.8250 | 97.2845 | 97.2746 |
| | Ghr [%] | - | 94.9298 | 71.2956 | 97.5467 | 97.5345 |

The confidence interval values reported in Tables 1 and 2 confirm significant differences in computational time between the new method and the other methods in all analyzed cases. These differences increase with growing problem complexity, as also evidenced by the simple individual indices, which show that the new method exhibits the lowest increase in computational time as a function of problem complexity.

In terms of the value of the optimization criterion, paired statistical tests demonstrated that the new method achieves statistically significantly lower values than the compared heuristic methods at all levels of complexity, both when solving the SGSP and when applied within the solution of the VRPR.

Table 3 presents the values of the statistical indicators obtained from the comparison of the new method with heuristic methods for solving the SGSP in terms of the optimization criterion value.

Table 2 Comparison of SGSP Solution Methods in Terms of the Optimization Criterion

| Problem complexity | | New method | MM | ZM |
|---|---|---|---|---|
| Low | Unsolved | 0 | 0 | 0 |
| | Gdr | - | 2.0780 | 8.9439 |
| | Ghr | - | 2.4654 | 10.3337 |
| Medium | Unsolved | 0 | 0 | 22 |
| | SII | 1.6422 | 1.6532 | 1.6856 |
| | Gdr | - | 2.7908 | 11.8838 |
| | Ghr | - | 3.1212 | 13.2315 |
| High | Unsolved | 0 | 0 | 99 |
| | SII | 2.3319 | 2.3644 | 2.7509 |
| | Gdr | - | 3.5253 | - |
| | Ghr | - | 3.7738 | - |

Table 4 presents the values of the statistical indicators obtained from the comparison of the new method with heuristic methods when applied within the solution of the VRPR in terms of the optimization criterion value.

Table 2 Comparison of VRPR Solution Methods in Terms of the Optimization Criterion

| Problem complexity | | New method | MM | ZM | MCW |
|---|---|---|---|---|---|
| Low | Unsolved | 0 | 0 | 0 | 0 |
| | Gdr | - | 0.6097 | 4.7844 | 4.9511 |
| | Ghr | - | 1.6633 | 6.9768 | 7.5016 |
| Medium | Unsolved | 0 | 0 | 0 | 1 |
| | SII | 1.1814 | 1.2063 | 1.3182 | 1.2098 |
| | Gdr | - | 2.2658 | 12.6519 | 7.4529 |
| | Ghr | - | 3.8123 | 16.4271 | 9.4333 |
| High | Unsolved | 0 | 0 | 73 | 83 |
| | SII | 1.5048 | 1.5897 | 2.0419 | 1.4760 |
| | Gdr | - | 5.5378 | 31.4458 | 9.9860 |
| | Ghr | - | 7.0891 | 38.1780 | 16.5347 |

The confidence intervals reported in Tables 3 and 4 confirm clear differences between the new method and the other compared methods in all analyzed cases, with these differences further increasing as problem complexity increases. This fact is also confirmed by the values of the simple individual indices, which indicate that the new method exhibits the lowest growth in the value of the optimization criterion as a function of problem complexity. The results further demonstrate that the new method maintains full problem solvability even at higher levels of complexity, whereas some of the compared methods gradually fail.

Due to its high computational efficiency, the proposed method allows the advantages of exact solutions to be exploited even for problems where this was previously time-inefficient, thereby contributing to more efficient EV routing for these problems and expanding the possibilities for their practical deployment.

## 4.2 Case study

The case study focused on the design of cyclic EV routes for the company Argos, operating in the retail distribution sector in the United Kingdom. The routes were designed with the aim of minimizing their total length.

The transportation network consisted of a depot, 40 customers, and five SDPs. Three time windows for customer service, a distance matrix, time-dependent travel time matrices, and a fleet of four electric vans with limited capacity, driving range, and the possibility of partial recharging along the route were considered.

The problem was solved using the NMCW method in combination with the new SGSP solution method. For comparison purposes, the problem was also solved using the NMCW method combined with the exact A* algorithm and with the MM heuristic, which were identified through statistical analysis as the most efficient exact and heuristic methods, respectively.

Figure 3 illustrates a comparison of the solutions generated by the individual methods in terms of total route length and computational time.

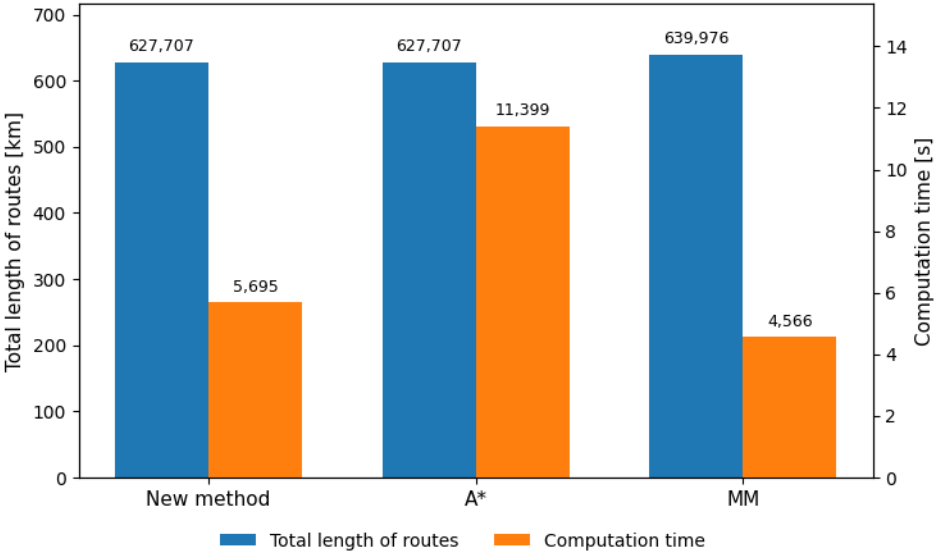


Figure 3 Comparison of Case Study Solution Methods

The results confirm the conclusions of the statistical analysis and show that the new method remains highly effective in terms of computation time and optimization criterion value even when applied in a real-world case study. At the same time, its flexibility is confirmed, as adapting it to the specific constraints and parameters of the case study did not require any major changes to the algorithmic structure. This confirms the method's potential for practical application and further development in the field of distribution problems.

# 5 Contributions of the Dissertation

The dissertation provides both theoretical and practical contributions in the field of distribution problems focused on EV routing and defines possible directions for further research.

## 5.1 Theoretical Contributions

The theoretical contribution of the dissertation is the creation of a new exact method for solving SGSP, which: 1) exploits novel dominance principles between nodes to minimize computational time compared to existing exact methods, 2) provides higher-quality solutions and a higher success rate in finding feasible routes compared to heuristic methods, 3) extends the practical applicability of exact solutions to computationally more complex problems, 4) is adaptable to the parameters of a specific problem, 5) exhibits increasing efficiency in terms of monitored indicators as problem complexity grows, compared to other methods, 6) is computationally efficient both when solving the SGSP as a standalone problem and when applied in the second phase of two-phase approaches to solving the VRPR.

## 5.2 Practical Contributions

The new method enables the practical application of exact solutions to the SGSP even in cases where heuristic methods had to be used due to time constraints. It is particularly useful when solving a large number of SGSP instances, typically within metaheuristic methods for solving the first phase of the VRPR. Furthermore, the method ensures optimal and feasible planning of EV energy replenishment stops, thereby expanding the possibilities for their use in distribution logistics. Practical applications of the method include in particular: 1) planning cyclic routes for large vehicle fleets, 2) solving dynamic distribution problems, 3) planning long-haul freight and passenger transportation, 4) solving shuttle and corridor trips.

## 5.3 Future Research Directions

Future research may focus on: 1) the development of two-phase methods for solving the VRPR that utilize the new method, particularly metaheuristic methods, 2) the development of a specialized software tool for solving the VRPR or the integration of the new method into existing platforms, 3) extending the new method to explicitly account for limited resources.

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# Abstract

The dissertation addresses the optimization of methods for solving distribution problems in logistics systems. Given the growing importance of electric vehicles in transportation, the work specifically focuses on electric vehicle routing problems and the planning of energy replenishment stops. The conducted literature review revealed that a key opportunity for improving solution methods in this area lies in reducing the computational time of exact methods for solving the Sequence Gas Station Problem, which is used to assign fuel or energy replenishment stations to routes with a given

sequence of serviced customers. Based on these findings, a new exact method for solving this problem was proposed in the dissertation. The effectiveness of the method was verified through statistical analysis, which demonstrated a statistically significant reduction in the computation time of the new method compared to the compared exact methods, on average by more than 99% in all analyzed cases. The method also exhibits a slower growth rate of computational time with increasing problem complexity. Compared to heuristic methods, the new method achieves better results in terms of both the value of the optimization criterion and the success rate of finding a feasible solution. The high computational efficiency of the new method allows the use of exact solutions even for problems where it was previously time-inefficient, thereby contributing to more efficient electric vehicle routing for these problems and expanding the possibilities for their practical application. The practical applicability of the new method was demonstrated through a real-world case study, which confirmed its high application potential and flexibility. Overall, the results of the dissertation represent an important step toward more efficient electric vehicle routing and open new opportunities for the development of optimization methods in distribution logistics.