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LOSS OF STABILITY OF THIN-WALLED CONICAL SHELLS WITH CIRCUMFERENTIAL RING LOADED BY AXIAL FORCE

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ABSTRACT

This dissertation is devoted to the effects of the circumferential ring on the loss of stability of the conical shells loaded by an axial force. The truncated conical shell with different shell thicknesses and base angles at the lower edge are investigated in this thesis. The main aim is a proposal a new method to calculation of load carrying capacity of the conical shell structures with a base angle less than 25° loaded by axial force. The proposed method is applicable for different radial stiffness of the circumferential ring. Two dimensionless similarity parameters are used in this method. Numerical models are created in COSMOS/M package program. The numerical analyses were performed for different angles, shell thickness and radial stiffness of circumferential ring. Empirical relationships are established based on the results of the numerical analysis.

KEYWORDS

Conical shell, Circumferential ring, Load Carrying Capacity, Axial loading, FEM

NÁZEV

Ztráta stability tenkostěnných kuželových skořepin s obvodovým prstencem zatížených osovou silou.

SOUHRN

Tato dizertační práce se zabývá vlivem obvodového prstence na ztrátu stability kuželových skořepin zatížených osovou silou. V této dizertační práci jsou zkoumány komolé kuželové skořepiny s rozdílnou tloušťkou pláště a úhlem vzepětí. Hlavním cílem je návrh nové metody pro výpočet únosnosti komolých kuželových skořepin s úhlem vzepětí nižším než 25° zatížených osovou silou. Navržená metoda je aplikovatelná pro různé radiální tuhosti obvodového prstence. V rámci této metody jsou použité dva bezrozměrné podobnostní parametry. Numerické modely jsou

vytvořeny v programu COSMOS/M. Numerické analýzy byly provedeny pro různé úhly vzepětí, tloušťky pláště skořepiny a radiální tuhosti obvodového prstence. Na základě výsledků numerických analýz jsou stanoveny empirické vztahy.

KLÍČOVÁ SLOVA

Kuželová skořepina, Obvodový prstenec, Únosnost, Osové zatížení, MKP

TABLE OF CONTENT

1 INTRODUCTION

Thin-walled shells have a widespread application in aerospace, mechanical, civil and structural engineering concepts in different shapes and types such as robots, shelters, domes, tanks, silos, machinery and energy absorbers. They have also significant importance for carrying liquids, pressurized gasses, and hazardous substances in road haulage, railroad and water transports. The use of the curved skin of vehicles as a load bearing member has similarly revolutionized the construction of aircraft. In the construction of all kind of spacecraft, the idea of a thin but strong skin has been used from the beginning. The demands in the thin-walled shells are quite prevalent as stated above. However, the thin-walled shells are considerably prone to loss of stability. Therefore, there is a great concern for the designers achieving maximum strength with a cost-efficient solution in the shells.

In present days, updated standards and recommendations provide useful approaches. They solve stability of the conical shells with the base angle which is higher than 25° and clamped lower end [5,6]. Nevertheless, the standard methods are not applicable for the shells which have the base angle less than 25°. Besides, the rules which are included in the recommendations can be applied only to conical shells which have clamped edges or edge with the very stiff ring. In other words, if a conical shell has either base angle less than 25° or free/flexible radial stiffness at the edges, these rules cannot be applied.

Determining the load carrying capacity of the nonstandard structure might be infeasible by referring the procedures within the context of the standards and recommendations because it is difficult to estimate the nonlinearity of the structure. Likewise, the recommendations and standard methods are based on the linear theory of the shells.

This study focuses on the load carrying capacity of the conical shells with a base angle less than 25° which have flexible boundary ring under axial loading. This area has lack of knowledge in the literature. Therefore, the main goal of the study is assigned to propose of a new method to estimate load carrying capacity of the conical shell structure with a base angle less than 25° for different radial stiffnesses under axial loading. The influence of the geometrical initial imperfection is included in the proposed method. A new reduction coefficient that simulates the effect of the initial imperfection on the load carrying capacity is suggested different from the standard and recommendations [5,6]. Thus, the load carrying capacity of the conical shells which stay in the non-linear area, that is mentioned above, can be estimated without any need of numerical analysis.

The study also aims to derive two dimensionless similarity parameters. These parameters allow for evaluation of the load carrying capacity of the conical shell for numerous configuration of geometrical dimensions in a wide range. One of these parameters represents the general geometrical form of the conical shell in terms of base angle, shell thickness, and radius. The other one characterizes the radial stiffness of the circumferential ring which is located around the lower edge of the conical shell.

Effects of the circumferential ring on the load carrying capacity of a conical shell has an important role. Effectiveness depends on the radial stiffness of the circumferential ring. It is quite indispensable to determine the contribution of the ring only by itself. The study also mentions the influence of the circumferential ring stiffness on the loss of stability. The boundary conditions are assigned according to fixed supported (infinite radial stiffness), simply supported (zero radial stiffness) and flexible radial stiffness at the lower edge. Numerical models and simulations have been performed using FEM package program COSMOS/M [9].

2 CURRENT SITUATION OF THE PROBLEM

Stability of the thin-walled shell structures has been studied by many prominent authors. Results of their studies are embedded in standards, regulations, and recommendations. In this dissertation, two of the most important documents of them are cited. These are "Recommendations for Design of Steel Shell Structures ECCS" [5] and the "European Standard for Design Steel Structures EN 1993-1-6-2007 [6]. On the standards and recommendations, limit state of the shell structures is given in consequence of buckling. Some of the other studies which are concerned in this dissertation study are presented by, Seide P., Chryssanthopoulos MK, Spagnoli A., Gupta NK and more.

3 SCOPE OF THE STUDY

Design criteria of the standard shell structures are provided with some analytical approximations within the context of standards and regulations. Whereas, designing of the nonstandard structures requires to perform numerical analysis or experiment.

Additionally, effects of the circumferential ring on the load carrying capacity of a conical shell have a significant role depending on the radial stiffness of the ring. It is quite indispensable to determine the contribution of the ring on load carrying capacity. It is a robust process to optimize a structure by understanding the individual influence of each design parameter.

As mentioned before, the conical shells with a base angle less than 25° behave highly nonlinear. Unfortunately, the limit load of these type of nonstandard structures is not determined in the literature, adequately. In addition to this, the standards and recommendations are not bright enough for conical shells with the low base angle which have flexible radial stiffness at the lower edge. Therefore, the standard methods are not applicable for the selected base angle and assigned boundary conditions in the study.

The main goal of this study is to propose a new method to estimate load carrying capacity of the conical shell structure with a base angle less than 25° for different radial stiffnesses under axial loading. The influence of the initial geometrical imperfection is included in the proposed method. A new reduction coefficient that simulates the effect of the initial imperfection on the load carrying capacity is suggested different from the standard and recommendations [5,6]. Thus, the load carrying capacity of the conical shells, which stay in the mentioned nonlinear area, can be estimated using the new proposed method without any need of numerical analysis.

The study also aims to derive two dimensionless similarity parameters. These parameters allow evaluation load carrying capacity of the conical shell for numerous configuration of geometrical dimensions in a wide range. Derivation of nondimensional similarity parameters makes possible to simulate real applications with a simple model. This study also aims to clarify the effect of radial stiffness of circumferential ring on the load carrying capacity under axial loading. For this reason, the study includes some topics as follows,

- Determination of the load carrying capacity of the conical shells which have a base angle of less than 25°.

- Investigation of the influence of the radial stiffness on the limit load.

- Evaluation of the effect of the initial geometrical imperfection on the load carrying capacity.

- Derivation of the dimensionless similarity parameters to evaluate limit load in a wide range of the conical shell geometries.

- Suggestion of a new methodology to estimate load carrying capacity for conical shells with a base angle less than 25°.

4 PROBLEM DESCRIPTION

4.1 Analytical Study

An analytical solution is a procedure for determining the carrying capacity of the thin-walled conical shell structures which is described in the standards and recommendations. This method is based on the linear shell theory, and it is derived from the critical load under axial loading of the cylinder. Amount of this value is adjusted with counting some coefficients for simulation of the real applications. These coefficients can originate from the effect of boundary conditions, geometry, material model, initial imperfection, etc. In order to adapt the analytical approach to the conical shells for evaluating the critical load, the existing regulations are modified by finding out compatible correlation factors.

4.2 Definition of the Model

Upper radius r_1 and bottom radius r_2 are defined 50 and 250 mm relatively for the simulations. The base angle is appointed as $10° \leq \alpha_c \leq 20°$. According to these values, the equivalent radius of the conical shell is set between 730 and 1440 mm. The width of the circumferential ring b_{ring} is chosen as a constant value which is 15 mm. Applying the load directly on the upper edge of the conical shell may cause convergence error in the numerical study. Stress gradients may occur on this line with the high amount of stress values. Therefore, a very stiff pipe is used on the upper edge to apply load. The load is distributed uniformly to the body of the structure by means of this stiff pipe (auxiliary surface). On the other hand, the conical shell, which is used as a connection component, is investigated in the study. In the present case, the stiff pipe also characterizes an adjacent part to simulate real condition more precisely. The height of the relatively stiff pipe h is assigned as 10 mm. Cross section area of the circumferential ring is evaluated between $6 \leq A_{ring} \leq 300$ mm^2.

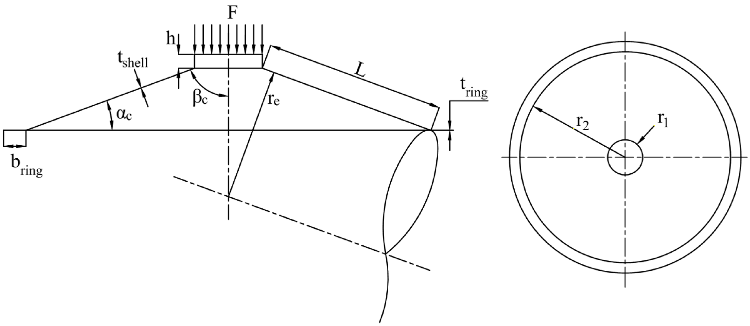

Figure 4.1. Front and top view of the conical shell (a) Front view (b) Top view

The thickness of the shell t_{shell} is set $0.5 \leq t_{shell} \leq 4\ mm$ interval. r_e/t_{shell} dimensionless parameter is assigned depending on the equivalent radius and the thickness of the shell between $240 \leq r_e/t_{shell} \leq 2880$. Additionally, the model is performed without ring (no radial stiffness) and with infinite stiff ring (fixed supported) in order to find ring effect on the limit load. Equivalent radius is calculated in the study as stated below,

$$r_e = \frac{r_2}{cos\beta_c} \qquad \text{Eq. 4.1}$$

where β_c is half-cone angle of conical shell in [Rad],

$$\beta_c = \frac{\pi}{2} - \alpha_c$$

Upper radius r_1 has relatively small effect on the carrying capacity of the conical shell structure under axial loading. It influences the capacity especially in the r_1/r_2 ratio is near to 1. These cases are not common in practical applications. The aim of the study is derivation non-dimensional parameter for similarity. To achieve this goal, it is needed to be a simplification. Otherwise the behavior characteristic of structure under axial load can solely be expressed as a partial function. This situation makes the problem highly complicated. For this reason, the r_1/r_2 ratio excluding between 0.1 and 0.8 is neglected and it is assumed that upper radius does not affect the limit load. The problem is simplified with a constant upper radius value (r_1=50mm). On the other hand, the value of r_2 is selected 250mm initially. But, it is used in similarity parameters as a variable that can be seen in further chapter of the study.

4.3 Numerical Study

Numerical models and simulations were performed using FEM programme COSMOS/M [9]. For the numerical analysis, large displacement module and Quadrilateral thick Shell element (SHELL4T) were assigned. Models were generated from three base angle α_c (10°, 15° and 20°). Basic sketch of the structure is illustrated in Figure 4.1. with dimension parameters.

In this study, geometrically nonlinear analysis (GNA) is performed using COSMOS/M [9], and the elastic limit load is carried out. At the first step of the study; the two limit conditions, which are fixed and simple end supported conical shells, are evaluated (Figure 4.2). It is important to see the extremities of the load carrying capacity.

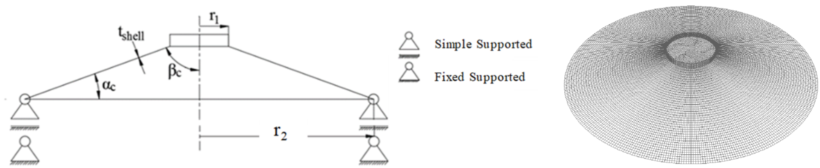

Figure 4.2. Schematic representation and numeric model of extreme cases.

In further studies, the limit load of the conical shell for various radial stiffnesses, which is represented by a circumferential ring, is investigated. The influence of the radial stiffness on the load carrying capacity is one of the central parts of this study. Schematic representation of the conical shell with the dimensions is illustrated in Figure 4.3 for this case. Models are axially loaded with 320 N at the initiation of the analysis. Because the upper edge is divided into 320 nodes and the load is assigned as 1 N for each node to make post-processing easy. The arc-length algorithm controls the loading increment step by step during the solution process.

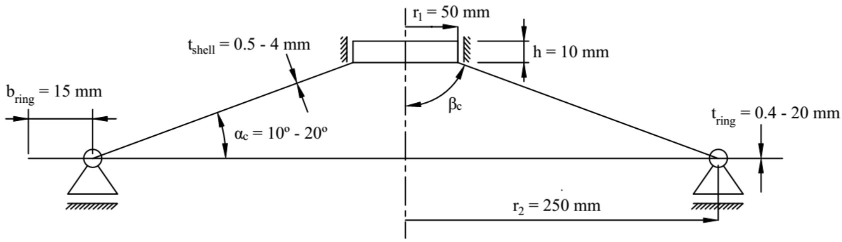

Figure 4.3. Schematic representation of the conical shell with the circumferential ring.

4.4 Mesh Study

A number of analyses are performed to evaluate dependency of the results to mesh structure. Thus, a conical shell which has current dimensions (Table 4.1) is modeled with different element sizes.

Table 4.1. Geometric dimensions and number of elements of the models (r_1=50 mm and r_2=250 mm).

Notation	$\propto_c$ [°]	t_{shell} $[mm]$	A_{ring} $[mm^2$	Γ	Number of element	F_{lim} $[kN]$	u $[mm]$
M_1	15	0.5	6	20.8	11002	6.778	3.72
M_2					20720	7.131	4.81
M_3					33308	7.101	4.18
M_4					49020	7.087	4.21

The load carrying capacity and the displacements in the vertical direction at the limit point can be seen in Figure 4.4 and Table 4.1. Displacement values are taken from same data point at which the top of the conical shell for all analysis.

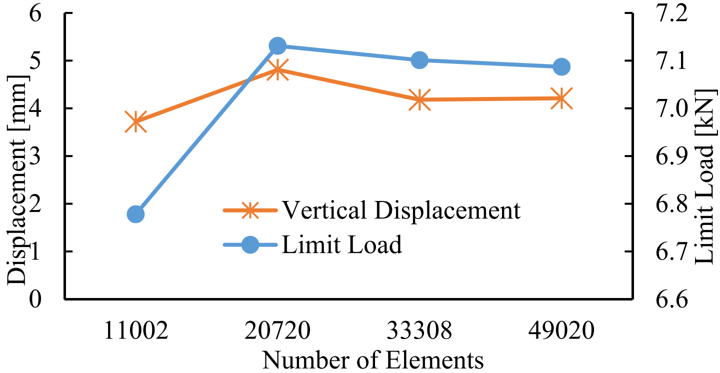

Figure 4.4. The results of the mesh study.

The results do not change over 1% between the models created with 2.6 mm (M_3) and 2.1 mm (M_4) element sizes. Therefore, the element size is chosen 2.6 mm for the numerical study considering computer supplement and time-consuming.

4.5 Boundary Conditions

In this study, the conical shells which are used as a connection component for the structure are investigated. Simple cone–cylinder connections are the most common form of connections and a ring is often provided to strengthen it. Therefore, a typical practical usage of the conical shell structures which have a circumferential ring with a cylindrical shell is modeled in the study (Figure 4.5). Assignment of the boundary conditions is a critical step for the numerical solution.to simulate problem, properly.

Concerning the solution time and mesh structure, the full scaled numerical model is simplified in the numeric analysis (Figure 4.6). This simplification provides decreasing the solution time and getting better mesh structure quality with lower computational system requirement. For this purpose, a simplified numerical model and a full scaled numerical model are performed to be able to make comparison. This step is vital to ensure whether the simplified model is simulated the full scaled one accurately or not.

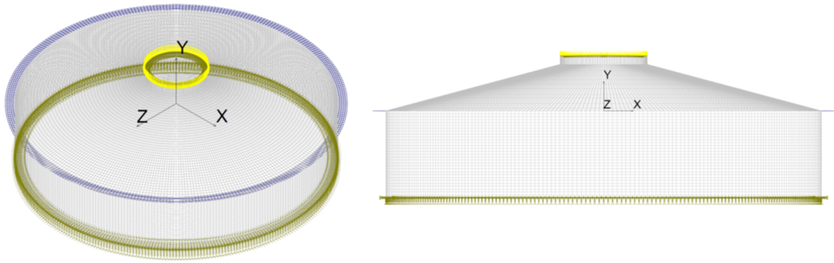

Figure 4.5. Full scaled numerical model.

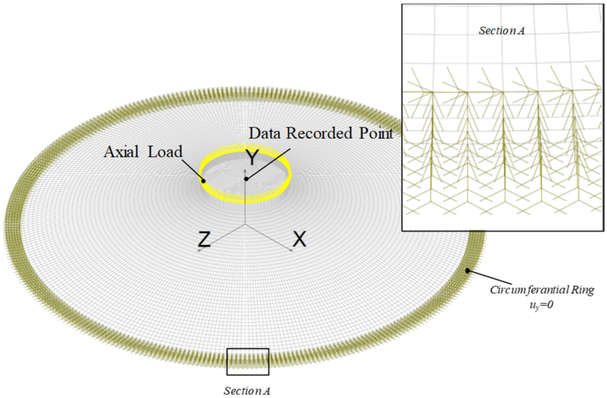

Figure 4.6. Simplified numerical model.

Four different models are generated, and they are illustrated with "*C_*" notation. The results show that the simplified models can simulate the full scaled model for various dimensionless parameters (r_e/t_{shell} and Γ. see Table 4.2).

Table 4.2. Geometric parameters and the limit load of the numerical models.

Notation	r_e/t_{shell}	Γ
C_1	1931.8	49.7
C_2	965.9	39.7
C_3	603.7	31.8
C_4	321.9	14.9

	$F_{lim}[N]$			
	C_1	***C_2***	***C_3***	***C_4***
Simplified. Model	7431.84	28498.88	72620.8	272617.6
Full Scaled Model	7494.72	29658.56	74268.8	272595.2

Limit loads of the conical shell structures are substantially same for different parameters. Thus, the simplified model is used instead of a full scaled model in the study hereinafter. This numerical model can be seen in Figure 4.6. The schematic representation of the conical shell with geometric parameters and boundary conditions are illustrated in Figure 4.3.

Where, the dimensionless Γ is the rigidity parameter of the circumferential ring. It depends on the radius of the lower edge of the conical shell, the thickness of the shell and the cross-sectional area of the ring. This parameter expresses the influence of the circumferential ring on load carrying capacity of the conical shells corresponding to the same base angle $\propto_c$. The parameter is represented in Eq. 4.2.

$$\Gamma = \frac{r_2 t_{shell}}{A_{ring}} \qquad \text{Eq. 4.2}$$

5 RESULTS AND DISCUSSION

The primary purpose of this chapter is to determine the carrying capacity of the conical shells under axial loading within two limit boundary conditions. In addition to this, the effect of the circumferential ring on the carrying capacity of the conical shell is investigated. These limit conditions simulate the ring allows unlimited radial displacement or entirely prevents against the radial displacement of the lower edge of the structure.

5.1 Influence of Boundary Conditions

The results of the numerical analysis for the conical shells show the limit load of the structure. The limit load is substantially dependent on selected boundary conditions (Figure 5.1). Possible displacement of the circumferential ring at radial direction causes a reduction in buckling strength of the structure.

Significance of the boundary condition against the carrying capacity of the conical shell increases, especially at the lower r_e/t_{shell} values. The results of the fixed supported conical shell (wholly restricted radial displacement) suggest that the circumferantial ring stiffness is quite efficacious on the limit load of the structure. The relation between the limit load and parameter r_e/t_{shell} of the conical shell is illustrated in Figure 5.1.

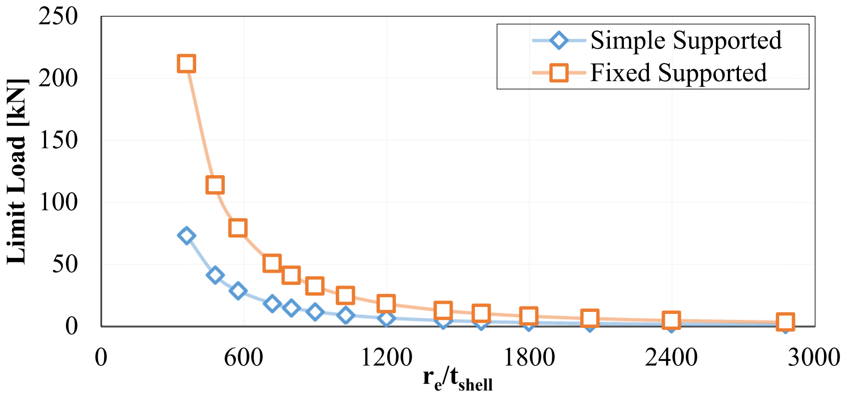

Figure 5.1. Influence of the boundary conditions on the carrying capacity of the conical shell base angle 10°.

It is possible to derive regression curve as a power function of this parameter using the data points (Eq. 5.1). The curve is relatively dependent on r_e/t_{shell} parameter.

$$F_{lim} = K'(r_e/t_{shell})^{-m'} \qquad \text{Eq. 5.1}$$

The Eq. 5.1 uses for creating the regression curves as a power function and data from GNA analyses are taken into consideration. Coefficients K' and m are shown in Table 5.1 considering the linear elastic behaviour of the fixed supported conical shell.

The load carrying capacitiy of the simple supported conical shell (utterly allowable radial displacement) is relatively low when compared to the fixed one at the same r_e/t_{shell} value. These differences are caused by the radial stiffness of the structures. That means, the carrying capacity of the structure is relatively dependent on the radial stiffness.

Table 5.1. The coefficients of the regression curves of the fixed supported conical shell.

Base angle $\propto_c$ [°]	Range of dimensionless parameter r_e/t_{shell}	Coefficients	
		K' [kN]	m'
10	480 - 2880	$3x10^7$	1.995
15	320 - 1930	$3x10^7$	1.999
20	240 - 1460	$3x10^7$	1.992

5.2 Conical Shell with Circumferential Ring

In the previous chapter, relations and coefficients were mentioned to calculate the carrying capacity of the simple supported and fixed supported conical shells. These boundary conditions at the lower edge are the representations of the extremities. However, in the practical application, the conical shell is used with the boundary conditions which are located between these two cases (with a circumferential ring). The conical shell is evaluated for the base angle $\propto_c = 10, 15, 20°$. Cross-sectional area of the circumferantial ring A_{ring} is kept between $6 \div 300\ mm^2$.

The following figure is drawn depending on the limit load for different circumferential ring stiffnesses. As expected, the curves which belong to various ring cross-sectional areas (different radial stiffness) are positioned between the extrimities. It is interesting that the ring area $A_{ring} = 6mm^2$ also contributes significantly positive effect to the carrying capacity of the conical shell. On the other hand, $A_{ring} = 300mm^2$ provides nearly same contribution with fixed supported one. Graph for base angles 10° is seen below (Figure 5.2). In addition to these, the limit load values can be seen in the section of appendixes which is found at the end of the dissertation.

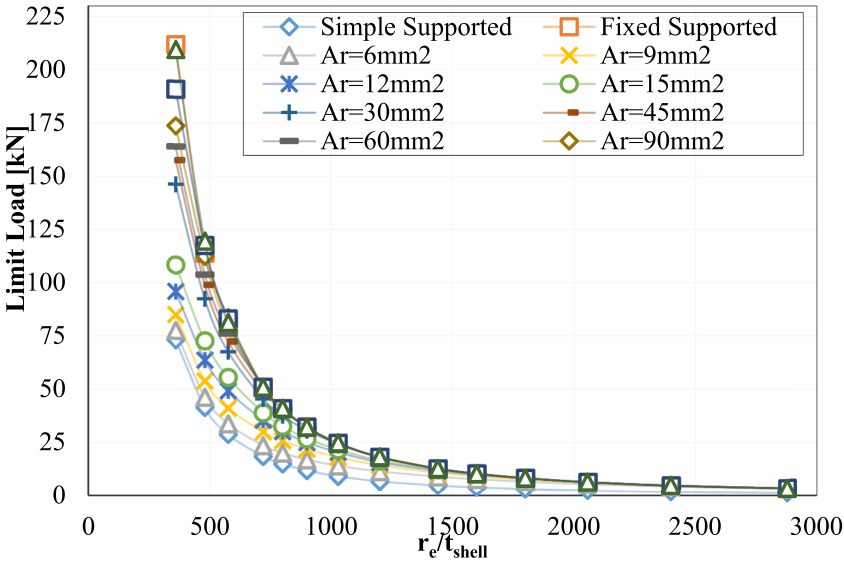

Figure 5.2. Limit loads for different radial stiffnesses.

It is apparently seen that the importance of the radial stiffness on the conical shell structures which have base angle lower than 25°, in Figure 5.2. The capability of load carrying can reach three times higher in comparison between the structures which have cross-sectional area of the circumferential ring of $A_{ring} = 300mm^2$ and $A_{ring} = 6mm^2$ in the lower r_e/t_{shell} ratios. However, this difference decreases in higher r_e/t_{shell} ratios. This situation is related to the slenderness of the structure. The expected limit load will be low in higher r_e/t_{shell} ratios, therefore, even the circumferential ring with $A_{ring} = 6mm^2$ behaves stiff enough against the radial displacement during the nonlinear collapse occurs. Hence, the limit loads of the structures with $A_{ring} = 6mm^2$ and $A_{ring} = 300mm^2$ become nearly same.

5.3 Similarity Criteria

The load carrying capacity of the conical shell which has r_1=50 mm and r_2=250 mm was investigated up to now in the present study. The influence of the geometrical parameters on the limit load of the structure was examined, separately. But, this section mentions about the derived similarity parameter which is the main aim of the thesis. Thus, the load carrying capacity of many different configurations of the conical shells can be estimated. For instance, a large conical shell which is used under operation can be simulated with a simple model using similarity parameters. In addition to this, the load carrying capacity of the structure can be calculated via Eq. 5.3 non-dimensionally without any need to a numerical analysis.

According to results, a similarity between load carrying capacities of the conical shells regarding geometrical parameters is tried to derive. Since distributed line load is applied to the structures, it is hard to express similarity in terms of limit load for different conical shell geometries. To achieve this purpose, the load is normalized by a constitutive relation with respect to the cross-section area of the lower edge. Therefore, normalized axial load (Eq. 5.2) is adapted to the results as exhibited in the literature before [44 and 45]. It is a function of limit load and geometrical parameters of the structure; besides, it represents the limit load in nondimensional form. The limit load of the structure can be calculated easily from this nondimensional form.

$$F_{Normalized} = \frac{F_{lim}}{2\pi r_2 t_{shell} E} \qquad \text{Eq. 5.2}$$

The function that is seen in Eq. 5.3 gives results in maximum 15% variation when it is compared to FEM. Normalized load can calculate with this equation using a and b from corresponding to base angle and rigidity parameter Γ.

$$F_{Normalized} = a\left({}^{r_e}/_{t_{shell}}\right)^{-b} \qquad \text{Eq. 5.3}$$

The aforementioned non-dimensional similarity parameters are r_e/t_{shell} and Γ. Γ is calculated as seen below. If these parameters are identical for the same base angle, the normalized load of these structures is expected to be equal.

$$\Gamma = \frac{r_2 t_{shell}}{A_{ring}}$$

The numerical analyses results and obtained values from Eq. 5.3 for randomly selected conical shell structures are seen in Table 5.2. The structures which are expected to operate in real applications have different upper and bottom radii.

Table 5.2 FEM and analytical results for the conical shells with base angle 10°

	$\boldsymbol{r_1}$ $[\boldsymbol{mm}]$	$\boldsymbol{r_2}$ $[\boldsymbol{mm}]$	$\boldsymbol{t_{shell}}$ $[\boldsymbol{mm}]$	$\boldsymbol{r_e}$ $\boldsymbol{/t_{shell}}$	**Γ**	$\boldsymbol{F_{Normaliz}}$ $\boldsymbol{*10^6}$ [-] (FEM)	$\boldsymbol{F_{Normalized}}$ $\boldsymbol{*10^6}$ [-] (Analytical (Eq. 5.3))
A_1	100	500	5	575.88	20	83.54	85.8
A_2	250	500	5	575.88	20	82.80	85.8
A_3	300	2000	20	575.88	20	86.81	85.8
A_4	800	2000	20	575.88	20	87.53	85.8
A_5	700	5000	50	575.88	20	86.95	85.8
A_6	2000	5000	50	575.88	20	88.04	85.8

It is seen that Eq. 5.3 has good agreement with the FEM results. Besides, the similarity parameters are well matched. The structures with various geometrical dimensions but same similarity parameters have a similar normalized load. In addition to this, if the rigidity parameter of the structure is not found in the Table 5.3, linear interpolation is used to get coefficients.

Table 5.3. Coefficients of the conical shell for parameter Γ.

Base Angle $\propto_c$ [°]	Range of r_e/t_{shell} parameter	Rigidity Parameter $\Gamma = \frac{r_2 t_{shell}}{A_{ring}}$	Coefficients a	b
10	480 - 2880	Fixed Supported	0.0696	0.995
		Simple Supported	0.0190	1.001
		1	0.1652	1.067
		5	0.1173	1.066
		10	0.0569	0.987
		20	0.0286	0.913
		40	0.0371	0.957
		60	0.0508	1.015
		80	0.0546	1.044
		100	0.0417	1.015
15	320 - 1930	Fixed Supported	0.1141	0.999
		Simple Supported	0.0289	0.998
		1	0.1697	1.032
		5	0.1320	1.028
		10	0.0814	0.979
		20	0.0424	0.899
		40	0.0515	0.948
		60	0.0700	1.008
		80	0.0730	1.032
		100	0.0614	1.025
20	240 - 1460	Fixed Supported	0.1526	0.991
		Simple Supported	0.0375	0.996
		1	0.2634	1.038
		5	0.2036	1.033
		10	0.1230	0.984
		20	0.0566	0.880
		40	0.0730	0.946
		60	0.0936	1.006
		80	0.0937	1.023
		100	0.0650	0.992

5.4 Influence of Initial Imperfection

When researching thin-walled shell structures, the influence of initial imperfections on the loss of stability cannot be ignored. The membrane stiffness of the shells is much higher than the flexural stiffness. Initial imperfections may cause bringing the structure into bending state at the beginning of loading.

The bending state may also arise due to the nature of the structure (e.g. conical shells with a base angle less than 25°). Therefore, the sensitivity of initial imperfection is less pronounced in nonstandard structures than the structures with membrane stress dominantly (e.g. a cylindrical shell).

Initial imperfections may be seen in several types, for example, imperfections in shape, structural attachment, non-uniform loading on the structure, residual stresses, uneven distributions of mechanical properties of the material, etc. One of the most notable imperfection is called initial geometrical imperfection. Initial geometrical imperfections may be caused during manufacturing or transportation of equipment.

Only the influence of geometrical initial imperfections on the load carrying capacity is investigated in the present study. The recommendation ECCS evaluates the initial geometrical imperfections as follows.

- Out of roundness
- Eccentricities
- Local dimples

Size of the characteristic imperfection amplitude Δw_k is measured using the ruler for measuring of initial geometric imperfections. The rulers are devised to relate to the size of buckles that are expected to form under each of the different basic load cases. The length of the ruler is determined by $l_g = \sqrt{4 r_2 t_{shell}}$ in axial loading (Figure 5.3). The influence of other geometrical imperfections (out of roundness and eccentricity) is less pronounced therefore, they are not investigated in this study.

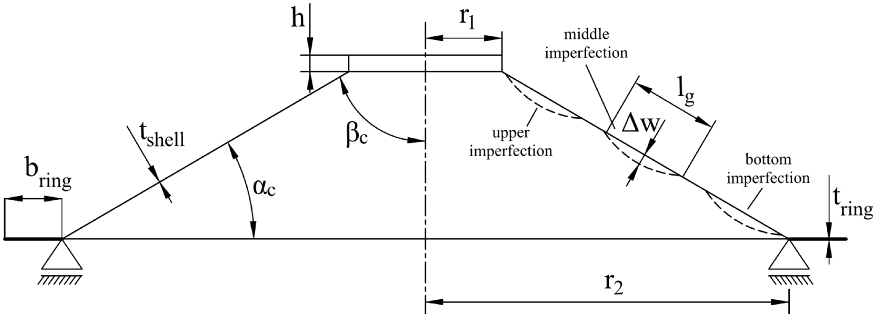

Figure 5.3. Characteristic imperfection amplitude (depth of dimple) and geometrical parameters symbolically.

The characteristic imperfection amplitude, Δw_k expresses the maximum permitted depth of the dimple. Its value depends on the quality of production and the geometrical dimensions of the conical shell.

$$\Delta w_k = \frac{1}{Q_{pr}} \sqrt{\frac{r_e}{t_{shell}}} t_{shell} \qquad \text{Eq. 5.4}$$

where, Q_{pr} is the influence of fabrication quality parameter from Table 5.4. The influence of the initial geometric imperfection on the load carrying capacity of the shell structure is expressed by the reduction coefficient α. In order to calculate the reduction coefficient, the ECCS [5] states the relationship as follows,

Table 5.4. Values of fabrication quality parameter Q_{pr} [5].

Fabrication tolerance quality class	Description	Q_{pr}
Class A	Excellent	40
Class B	High	25
Class C	Normal	16

$$\alpha = \frac{0.62}{1 + 1.91(\Delta w_k / t_{shell})^{1.44}} \qquad \text{Eq. 5.5}$$

Changing of the reduction coefficient α is shown in Figure 5.4 depending on r_e/t_{shell}. The figure apparently shows that the structure with higher r_e/t_{shell} parameter is more sensitive to initial geometrical imperfection where the reduction coefficient α decreases.

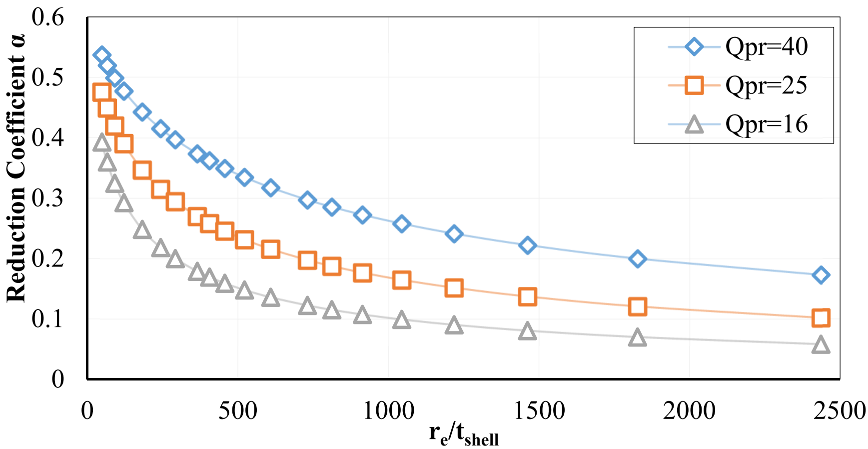

Figure 5.4. The change of the reduction coefficient.

The reduction coefficient is presented in the ECCS for a fixed supported conical shell (infinite radial stiffness) under axial loading. However, the bending effect is higher for simple supported conical shell than a fixed supported conical shell. Therefore, it can be assumed that the reduction coefficient specified in the ECCS is too conservative for the simple supported structure. For this reason, it is important to determine the dependence of the reduction coefficient on the radial stiffness of the conical shell. Figure 5.5 shows the dependence of reduction coefficient on the depth of imperfection Δw

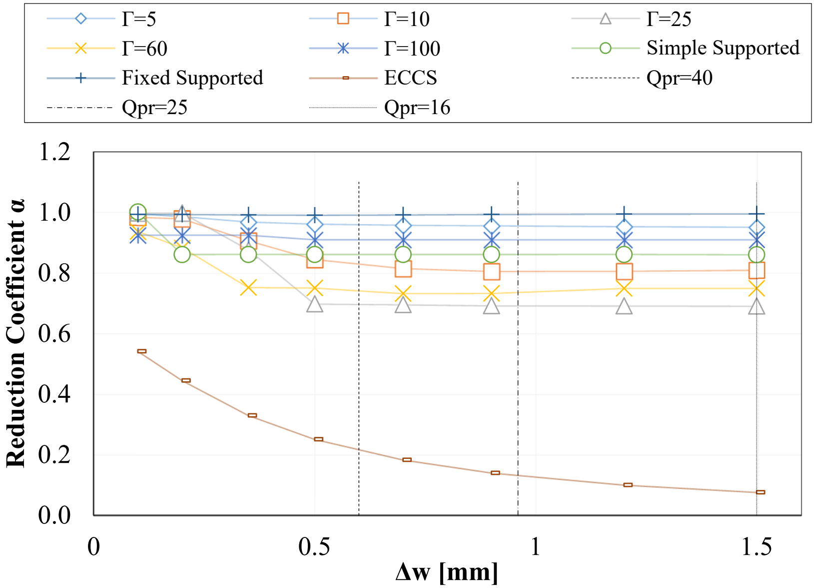

Figure 5.5. Reduction coefficient for different configuration of conical shells with a base angle 15°.and t_{shell} = 0.6 mm.

The production quality classes (Table 5.4) and their respective reduction coefficients according to recommendation [5] are illustrated in the graphs (vertical lines nominated with Q_{pr}). Those lines also show the maximum permissible depth of the imperfection. The reduction coefficient values that obtained from ECCS are too conservative for conical shells even for the quality class A. Hence, this value could be replaced by $\alpha = 0.70$.

It is also interesting to see from the curves in Figure 5.5 that the increment in the depth of imperfection can result in a reduction of its effect on the load carrying capacity. The reduction coefficient may tend to increase after a point in some cases. In those cases, the dimple starts to act as a stiffener.

According to the European Recommendation, the value of the reduction coefficient for conical shells under axial loading is calculated using Eq. 5.5. The proposed calculation methodology by ECCS to estimate load carrying capacity of the conical shells is originated from the cylindrical shells. In contrast to the cylindrical shells, the conical shells have a significant bending stress. Therefore, the influence of initial imperfections, which represents an additional bending effect, is less significant in conical shells with base angle less than 25°. From this point of view, it can be assumed that the reduction coefficient that is determined based on the cylindrical shell is considerably conservative for conical shells. The reduction coefficient does not drop below $\alpha = 0.70$ for different combinations of shell thickness and radial stiffness. Thus, the new value of the reduction coefficient is determined in this chapter as a constant value of $\alpha = 0.70$.

6 COMPARISON OF CALCULATION METHODS

The load carrying capacity estimations for the conical shells are compared in this chapter. Obtained results from the ECCS recommendations, the proposed method in the present study and the numerical analyzes are examined. The load carrying capacity values of the conical shells with diverse types of boundary conditions are considered. Influence of the initial imperfections is taken into account using the reduction coefficient that is proposed as α=0.70 in previous section.

The dimensions of the fixed conical shell are shown in Table 6.1. The geometry of the cylinder, which originates from the geometry of the conical shell, falls within the area of medium length cylindrical shells. For this reason, the influence of boundary conditions, which is expressed by the C_x factor, assigned to 1. It is assumed that the loss of stability occurs in the elastic region for the randomly selected conical shells. Therefore, the effect of the elastic-plastic behavior of the material is not applied for the solutions. The reduction coefficient is determined according to the ECCS recommendation by a_x in calculation of ECCS (Eq. 5.5). (for reasons of clarity, the reduction coefficients are indicated with the corresponding indices as a_{NM} and a_{ECCS} in the following sections).

Table 6.1. Fixed supported conical shell dimensions.

a_c[°]	r_1[mm]	r_2[mm]	t_{shell}[mm]	r_e[mm]	r_e/t_{shell}[-]	C_x[-]
10	200	600	2	3455	1728	1

Calculation according to ECCS

Critical elastic stress for the conical shell is expressed by the relation,

$$\sigma_{xRcr} = 0.605EC_x\frac{t_{shell}}{r_e} = 0.605x2E5x1x\frac{2}{3455} = 70[MPa]$$

Characteristic imperfection depth for production quality class A and the reduction coefficient are determined as follows,

$$\Delta w_k = \frac{1}{Q_{pr}}\sqrt{\frac{r_e}{t_{shell}}}t_{shell} = \frac{1}{40}\sqrt{\frac{3455}{2}}2 = 2.08\ [mm]$$

$$\alpha_{ECCS} = \frac{0.62}{1 + 1.91(\Delta w_k/t_{shell})^{1.44}} = \frac{0.62}{1 + 1.91(2.08/2)^{1.44}} = 0.21$$

The factor C_x is determined using the relationship for a medium length cylindrical shell regarding to the dimensionless length parameter ω.

$$\omega = \frac{l_e}{\sqrt{r_e t_{shell}}}$$

It is assumed that the loss of stability will occur for this conical shell structure in the elastic area (i.e. $\lambda_x \geq \lambda_p$). The characteristic buckling stress is given by the relation,

$$\sigma_{xRk} = \chi_x f_{y,k} = \frac{\alpha}{\lambda_x^2} f_{y,k} = \alpha_{ECCS}\sigma_{xRcr} = 0.21x70$$
$$= 14.7\ [MPa]$$

Limit load is calculated with the following equation for the conical shell,

$$F_{lim,ECCS} = 2\pi r_e t_{shell}\sigma_{xRk}cos^2\beta_c$$
$$= 2x\pi x3455x2x14.7xcos^2(80) = 19245[N]$$

then the limit load is normalized using geometrical parameters and modulus of elasticity,

$$F_{Normalized,ECCS} = \frac{F_{lim,ECCS}}{2\pi r_2 t_{shell}E} = \frac{19245}{2x\pi x600x2x2E5}$$
$$= 12.76x10^{-6}\ [-]$$

GNA type numerical analysis

Load carrying capacity of the conical shell (see Table 6.1) obtained using GNA type numerical analysis. The numerical result is seen below. The influence of initial imperfections is taken into account by means of the proposed reduction coefficient value α_{NM}=0.70.

$$F_{lim,GNA} = 68171.5\ [N]$$

when the effect of initial imperfections is considered the limit load decreases to $F_{lim,GNA,\alpha}$,

$$F_{lim,GNA,\alpha} = \alpha_{NM}F_{lim,GNA} = 0.70x68171.5 = 47720\ [N]$$

and it is normalized,

$$F_{Normalized,GNA,\alpha} = \frac{F_{lim,GNA,\alpha}}{2\pi r_2 t_{shell}E} = \frac{47720}{2x\pi x600x2x2E5}$$
$$= 31.64x10^{-6}\ [-]$$

Proposed method

The proposed method calculates the load carrying capacity by means of Eq. 5.3 as given below,

$$F_{Normalized} = a\left({r_e}/{t_{shell}}\right)^{-b}$$

The coefficients of the regression curve of the fixed supported conical shell are shown in Table 5.3.

Table 6.2. Regression curve coefficients

a	b
0.0696	0.995

$$F_{Normalized} = a\left({r_e}/{t_{shell}}\right)^{-b} = 0.0696\left({3455}/{2}\right)^{-0.995} = 41.82x10^{-6}\ [-]$$

and the effect of initial imperfections reduces the calculated value using reduction coefficient α_{NM},

$$F_{Normalized,\alpha} = \alpha_{NM}F_{Normalized} = 0.70x41.82x10^{-6} = 29.28x10^{-6}\ [-]$$

Comparison of the results

A summary of the previous results is shown in Table 6.3.

Table 6.3. Comparison of results for the fixed supported conical shell.

Fixed Supported Conical Shell	**ECCS**	**GNA Analysis**	**Proposed Method**
$F_{Normalized}$***10**^{6} **[-]**	12.76	31.64	29.28

The value of the reduction coefficient α_{ECCS} is relatively low in the calculation of ECCS. Thus, the calculation with respect to ECCS is quite conservative in the case of the fixed supported conical shell. On the other hand, the proposed method gives quite compatible value to numerical result.

7 CONCLUSION

In this study, the load carrying capacity of the conical shell structures which have different radial stiffnesses is examined. The base angle of the conical shell structures is kept less than 25° to contribute to filling the deficiency in the literature. A new method is proposed to estimate the load carrying capacity for mentioned conical shell structures. Results which are obtained from the nonlinear FEM analyses are stated below.

In order to predict load carrying capacity of the conical shell structures under the axial load with lower base angles (i.e. 10, 15 and 20°), nondimensional design parameters (Γ and r_e/t_{shell}) are derived. Based on these parameters, a similarity approach is proposed which estimates load carrying capacity of the shells of different shell geometry configurations at the same base angle. This similarity approach tells that the two different shell configurations having the same Γ, r_e/t_{shell} and base angle have the same normalized loads. Practically, this provides an enormous advantage of estimating load carrying capacity of the conical shells from small to large structures. Therefore, there is no need to perform some series of the experiments to determine the load carrying capacity of the structures.

A simple expression is proposed to calculate the normalized load of the conical shell structure as a function of the dimensionless geometrical shell parameters and two constant coefficients of “a” and “b” which are selected considering the base angle, rigidity parameter. In this way, it enables an appropriate prediction of the load carrying capacity of the conical shell structures under the axial load for a variety of the shell configurations without performing some complex non-linear FEM analysis or numerical solutions. Furthermore, the discrepancy of the proposed new method and FEM results of the normalized load is found out to be the maximum

15% which can be considered in the acceptable limits for a highly nonlinear shell behavior of the lower base angles (10, 15 and 20º).

Implementation of the linear theory in the load carrying capacity calculations concludes with the high amount of deviations due to the presence of the circumferential ring and highly nonlinear shell response of the shell structures which is encountered at low base angles such as 10, 15 and 20º. The proposed expression for the normalized load minimizes this aforementioned deviation and keeps the results within the acceptable limits. Since particular equation coefficients of "a" and "b" are selected in order to characterize the non-linear response of the corresponded shell geometry.

In the scope of the thesis, the value of r_1 is kept constant to be 50mm. Because the influence of the upper shell radius r_1 on the load carrying capacity can be neglected for a wide range of upper-to-bottom shell radius ratios "r_1/r_2" as a result of performed FEM simulations. However, the influence of the upper shell radius on the load carrying capacity of the shell structure is observed to be more apparent as the upper-to-lower shell radius ratio "r_1/r_2" approaches its extremities which are $r_1/r_2=0$ and 1.

The influence of initial imperfections, which represents an additional bending effect, is less significant in conical shells with base angle is less than 25°. From this point of view, it can be assumed that the reduction coefficient that is determined based on the cylindrical shell is considerably conservative for conical shells. The reduction coefficient does not drop below $\alpha = 0.70$ for different combinations of shell thickness and radial stiffness. Thus, the new value of the reduction coefficient is proposed as a constant value of $\alpha = 0.70$.

Circumferential ring implementation and its radial stiffness make a contribution to the load carrying capacity of the structure under the axial loading. The influence of the radial stiffness increases as r_e/t_{shell} parameter decreases. The results show that application of circumferential ring in lower r_e/t_{shell} dimensionless parameter value becomes more advantageously.

7.1 Scientific Contribution of the Doctoral Dissertation

The influence of the circumferential ring on the load carrying capacity of a conical shell under axial loading has not been involved fully in the European Recommendation ECCS [5]. Also, relationships in the recommendation are not applicable to the conical shells structures that have a base angle less than 25°.

Evaluation of the load carrying capacity of the fixed supported shell structures is outlined in the ECCS [5]. Nevertheless, this calculation may give higher values for the real application because of the flexible radial restrains. The results are presented in the study in order to complement the current state of knowledge of science and technology.

Determination of the limit load for the nonstandard conical shell structures with a circumferential ring has not been resolved yet. Validating the numerical results with the experimental study is necessary. After validation process, the study will be put on authorities display.

7.2 Implementation of the Results in Practice

This study proposes a new method to predict load carrying capacity of a conical shell structure and it suggests similarity parameters. By means of these parameters, experiments can be performed with small-scaled structures for simulation the real one. Additionally, the method allows estimation the load carrying capacity without any need of numerical analysis and avoids time-consuming. Thus, efforts have been made to contribute to the design process in fields such as transport, machinery and civil engineering where thin-walled shells are widely used. For this purpose, further studies should be accomplished primarily.

7.3 Future Works

For the further parts of the current study, the following evaluations and statements are to be completed, respectively which are;

- A validation methodology will be conducted in order to ensure that how the numerical study approaches the experimental results. Hereby, a specimen will be manufactured, and it will be loaded by a hydraulic press. A load history concerning vertical deformation will be extracted to make a comparison of proximity.
- The influence of the material nonlinearity is described in the ECCS for cylindrical shells. It is suggested that the determination of the transition boundaries to the plastic and elastic-plastic region for the conical shells under axial loading should be investigated in further studies.

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